CHAPTER 1
INTRODUCTION

Research may start from definite problems whose importance it recognizes and whose solution is sought more or less directly by all forces. But equally legitimate is the other method of research which only selects the field of its activity and, contrary to the first method, freely reconnoitres in the search for problems which are capable of solution. Different individuals will hold different views as to the relative value of these two methods. If the first method leads to greater penetration it is also easily exposed to the danger of unproductivity. To the second method we owe the acquisition of large and new fields, in which the details of many things remain to be determined and explored by the first method. A Clebsch [1871, p. 6]

The theory of screws was first established by Ball [1900] after a quarter century of development. Although most mathematicians of the late nineteenth century were acquainted with screw theory, very few actually pursued its development. Consequently, in the early twentieth century it virtually faded into obscurity. However, with a revived interest in spatial mechanisms in the 1960s and robotics in the 1970s, screw theory is being slowly resurrected in either the guise of dual vectors or Plucker coordinates. An excellent treatment of Plucker coordinates in application to mechanisms has been given by Woo and Freudenstein [1970]. Recent texts by Hunt [1978] and Bottema and Roth [1979] have investigated and applied screws in various kinematic areas.

However, the problem that was initially investigated for this dissertation, the "orthogonality" of screws, was not treatable using the current literature in the field of kinematics. It was necessary to reconsider some of the geometrical work of the nineteenth century and try to put it in a modern perspective with improvements of notation, methods, etc. The beginning of this dissertation deals with a systematic development of the geometry of points, planes and lines to provide a broad basis for understanding the subtleties of screw theory in a unified framework. As such, the author has tried to integrate some of the pertaining accomplishments of the great late nineteenth century geometers: Ball, Cayley, Clifford, Grassmann, Klein and Plucker. Whenever possible, original papers were referred to in order to understand the spirit and implications between the lines which are forever lost in most modern presentations. Original quotes have been included throughout to compensate somewhat for this lamentable circumstance.

The geometrical development rightly begins with projective geometry, which is the most general, in Chapter 2. In order to commence on an analytical basis at the outset, coordinates are introduced in a metric-free manner for points. They are then generalized using determinant principles which elegantly display dualistic properties. Projective transformations leave incident relations invariant and are developed based on a tetrahedron principle.
metrical geometry is developed in Chapter 3 from projective geometry based on Cayley's conception of the Absolute. Elliptic geometry is detailed with emphasis on determining elliptic relations for screws that appear similar to projective ones. Here the elliptic polarity is first introduced. Elements of Euclidean geometry are then derived with what appears to be a new contribution in deducing Euclidean collineations based directly on the invariance of the Absolute. Vectors are introduced and are used to characterize screws in particular.

Chapter 4 deals with the role of the elliptic polarity in Euclidean space. It uses the previous developments to explain the motivation for and the importance of this work. Unless specifically noted all developments in this chapter are claimed to be original. A series of new mappings of screws is introduced, central to which is a quaternion representation. Ball's planar representation of the two-system of screws is generalized as a mapping on an inversive plane where the elliptic polarity induces a conformal mapping.

In Chapter 5 screw theory is applied to the hybrid control of robotic manipulators. It is demonstrated that a current theory based on "orthogonal" projection yields noninvariant results under Euclidean translations and changes in the unit of length. By way of introducing invariant kinematic filters, new invariant methods of hybrid control are presented.